Math 211 - Bonus Exercise 2 (please discuss on Forum)

1) Show that the any finite group of even order has an element of order 2.

2) Find a formula for the order of an element of S_n in terms of its cycle decomposition (i.e. as a product of disjoint cycles).

3) Try to find a minimal set of generators of S_n, i.e. elements $\sigma_1, \ldots, \sigma_k \in S_n$ with k as small as possible, such that there is no proper subgroup of S_n which contains all these elements.

4) Prove that there is no isomorphism of groups $\mathbb{Z} \cong \mathbb{Q}$ and $\mathbb{Q} \cong \mathbb{R}$ (all groups with respect to addition). For the last question, please don't use hard logic stuff like "countable" vs "uncountable" sets.

5) Recall the action of S_4 on a cube (or octahedron, if you prefer) by rotations. Describe the stabilizer subgroup of a vertex, edge, and face, respectively. By this I mean to think about the resulting action of S_4 on the set of vertices, the set of edges, and the set of faces, respectively, and describe the stabilizer subgroup of a given element of the aforementioned sets.